Comparing the (1+1)-CMA-ES with a Mirrored (1+2)-CMA-ES with Sequential Selection on the Noiseless BBOB-2010 Testbed

Anne Auger, Dimo Brockhoff, Nikolaus Hansen

Comparing the (1+1)-CMA-ES with a Mirrored (1+2)-CMA-ES with Sequential Selection on the Noiseless BBOB-2010 Testbed

Anne Auger, Dimo Brockhoff, and Nikolaus Hansen
Projet TAO, INRIA Saclay—Ile-de-France
LRI, Bât 490, Univ. Paris-Sud
91405 Orsay Cedex, France
firstname.lastname@inria.fr

ABSTRACT

In this paper, we compare the (1+1)-CMA-ES to the $(1+2_{\rm m}^{\rm s})$-CMA-ES, a recently introduced quasi-random (1+2)-CMA-ES that uses mirroring as derandomization technique as well as a sequential selection. Both algorithms were tested using independent restarts till a total number of function evaluations of 10^4D was reached, where D is the dimension of the search space. On the non-separable ellipsoid function in dimension 10, 20 and 40, the performances of the $(1+2_{\rm m}^{\rm s})$-CMA-ES are better by 17% than the best performance among algorithms tested during BBOB-2009 (for target values of 10^{-5} and 10^{-7}). Moreover, the comparison shows that the $(1+2_{\rm m}^{\rm s})$-CMA-ES variant improves the performance of the (1+1)-CMA-ES by about 20% on the ellipsoid, the discus, and the sum of different powers functions and by 12% on the sphere function. Besides, we never observe statistically significant results where the $(1+2_{\rm m}^{\rm s})$-CMA-ES is worse than the (1+1)-CMA-ES.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

The (1+1)-CMA-ES is an elitist version of the CMA-ES algorithm [10] that uses the (1+1) selection scheme where at each iteration, an offspring (new probe point) is created from the *single* parent (current solution) and the best among the offspring and parent is selected for becoming the new parent at the next iteration [11]. The (1+1)-CMA-ES implementing an independent restart mechanism was benchmarked for the BBOB-2009 workshop on the noiseless and noisy testbed [3, 4]. The overall best performing algorithm of the BBOB-2009 workshop [8] was the BI-POP variant of the CMA-ES algorithm, combining restarts of CMA-ES with large and small population sizes [6]. Surprisingly, the (1+1) variant of CMA-ES could outperform the BI-POP-CMA-ES algorithm by a significant factor on the Gallagher functions f_{21} and f_{22} [2]. On f_{21}, the (1+1)-CMA-ES is 8.2 times (resp. 68.7 times) faster than the BI-POP-CMA-ES in dimension 20 (resp. 40); for f_{22}, the (1+1)-CMA-ES is 37 times faster than the BI-POP-CMA-ES in 20D and is able to solve the problem in 40D which the BI-POP-CMA-ES does not allow.

Motivated by this surprisingly large improvement over the BI-POP-CMA-ES, new local search variants of CMA-ES have been designed combining derandomization by means of mirroring and sequential selection [1]. In this paper, we benchmark the elitist variant of these newly introduced algorithms, namely the $(1+2_{\rm m}^{\rm s})$-CMA-ES, and compare it to the (1+1)-CMA-ES.

2. THE ALGORITHMS

2.1 The (1+1)-CMA-ES

The (1+1)-CMA-ES with independent restarts used in this paper is the same as the one used in [3] and we refer to this paper for a thorough description of the algorithm and of the parameter setting. Because the function instances changed for the BBOB-2010 workshop, we ran the experiments again using the same parameters and stopping criteria. No parameter tuning per function has been done such that the crafting effort CrE equals 0.

2.2 The $(1+2_{\rm m}^{\rm s})$-CMA-ES

The $(1+2_{\rm m}^{\rm s})$-CMA-ES is a derandomized variant of the (1+2)-CMA-ES that, moreover, employs a sequential selection [1]. The derandomization of the (1+2)-CMA-ES is done by mirroring the two offspring with respect to the parent: let us denote $\boldsymbol{X}_n \in \mathbb{R}^D$, σ_n and $\boldsymbol{C}_n$ the current parent solution, step-size and covariance matrix respectively at iteration n. The first offspring equals $\boldsymbol{X}_n^1 = \boldsymbol{X}_n + \sigma_n \mathcal{N}_1(\mathbf{0}, \boldsymbol{C}_n)$ where $\mathcal{N}_1(\mathbf{0}, \boldsymbol{C}_n)$ is a random vector sampled from a multivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{C}_n$. The second offspring is the symmetric of $\boldsymbol{X}_n^1$ with respect to $\boldsymbol{X}_n$ and thus equals $\boldsymbol{X}_n^2 = \boldsymbol{X}_n - \sigma_n \mathcal{N}_1(\mathbf{0}, \boldsymbol{C}_n)$. Instead of selecting the best among $\boldsymbol{X}_n^1$ and $\boldsymbol{X}_n^2$ to become the next parent $\boldsymbol{X}_{n+1}$, as it is done in a (1+2) selection scheme, we compute the objective function value of $\boldsymbol{X}_n^1$ and compare it to $\boldsymbol{X}_n$, if $f(\boldsymbol{X}_n^1) \leq f(\boldsymbol{X}_n)$[1] we set $\boldsymbol{X}_{n+1} = \boldsymbol{X}_n^1$ and continue with the next iteration, else we compute the objective function value of $\boldsymbol{X}_n^2$ and set $\boldsymbol{X}_{n+1}$ to the argmin of $f(\boldsymbol{X}_n^1), f(\boldsymbol{X}_n^2)$ and $f(\boldsymbol{X}_n)$ as in the (1+2) selection scheme where $f : \mathbb{R}^D \to \mathbb{R}$ is the objective function to be minimized.

The notation for the parameters used are the one from [3]. We have used for the step-size control $d = 1 + \frac{D\lambda}{2}$, $p_{\text{target}}^{\text{succ}} = \frac{1}{5+\sqrt{\lambda/4}}$, $c_p = \frac{\lambda \cdot p_{\text{target}}^{\text{succ}}}{(2+\lambda \cdot p_{\text{target}}^{\text{succ}})}$ with $\lambda = 2$. For the covariance matrix adaptation, we have used $p^{\text{thresh}} = 0.44$, $c_{\text{cov}} = \frac{2}{D^2+6}$ and $c_c = \frac{2}{D+2}$. Moreover, an independent restart mechanism has been implemented for the (1+2_{m}^{s})-CMA-ES using the same stopping criteria as for the (1+1)-CMA-ES (see [3]). Each initial solution $\boldsymbol{X}_0$ was uniformly sampled in $[-4, 4]^D$ and the step-size σ_0 was initialized to 2. The source code used for the experiments is available[2].

As for (1+1)-CMA-ES, the crafting effort of (1+2_{m}^{s})-CMA-ES equals 0.

3. CPU TIMING EXPERIMENTS

For the timing experiment, both algorithms were run on f_8 with a maximum of $10^4 \times D$ function evaluations and restarted until at least 30 seconds have passed (according to Figure 2 in [7]). The experiments have been conducted with an 8 core Intel Xeon E5520 machine with 2.27 GHz under Ubuntu 9.1 linux and Matlab R2008a. The time per function evaluation was 5.3; 5.3; 5.4; 5.9; 7.1; 12 times 10^{-4} seconds in dimensions 2; 3; 5; 10; 20; 40 respectively for the (1+1)-CMA-ES and 4.6; 4.7; 4.7; 5.0; 5.3; 7.4 times 10^{-4} seconds in dimensions 2; 3; 5; 10; 20; 40 respectively for the (1+2_{m}^{s})-CMA-ES. Note that MATLAB distributes the computations over all 8 cores only for 20D and 40D.

4. RESULTS

Results from experiments according to [7] on the benchmark functions given in [5, 9] are presented in Fig. 2, 3 and 4 and in Table 1. Moreover, Fig. 1 shows results of the (1+2_{m}^{s})-CMA-ES function by function.

The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_{\text{t}} = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_{t}, summed over all trials and divided by the number of trials that actually reached f_{t} [7, 12]. **Statistical significance** is tested with the rank-sum test for a given target Δf_{t} (10^{-8} in Figure 2) using, for each trial, either the number of needed function evaluations to reach Δf_{t} (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration. We refer to a test with a p-value of $p \leq 0.05$ as statistically significant, and if $p \leq 10^{-3}$ as statistically highly significant.

When comparing the (1+1)-CMA-ES and the (1+2_{m}^{s})-CMA-ES, three main observations are worth to mention. First, the (1+2_{m}^{s})-CMA-ES is never statistically significantly slower than the (1+1)-CMA-ES. Second, in case both algorithms solve a function and a statistically significant difference can be observed, i.e., on functions f_1, f_2, f_{10}, f_{11}, and f_{14} (in 20D and for a target of 10^{-7}), the (1+2_{m}^{s})-CMA-ES is 10–20% faster than the (1+1)-CMA-ES. And third, on f_{11} in 5D (target value 10^{-7}), and on f_{10} (target values 10^{-3}, 10^{-5}, 10^{-7}) and f_{14} in 20D (target value 10^{-7}), the (1+2_{m}^{s})-CMA-ES beats the best result obtained in the BBOB-2009 benchmarking for those functions [2]. In particular on the non-separable ellipsoid function (f_{10}), where the best algorithm in the BBOB-2009 benchmarking was the (1+1)-CMA-ES for target values of 10^{-5} and 10^{-7}, this difference is statistically highly significant—improving the best algorithm of BBOB-2009 on this function in dimensions 10, 20 and 40 by 17% (for target values of 10^{-5} and 10^{-7}), cp. Table 1 but also Fig. 1. Note that on f_{10}, the factor of the (1+1)-CMA-ES in Table 1 is not exactly 1 as the experimental procedure changed from 5 instances per function with 3 runs each in 2009 to 15 independent instances in 2010 and we, therefore, rerun the experiments.

According to Fig. 4, the biggest impact of the sequentialism and mirroring in the (1+2_{m}^{s})-CMA-ES can be seen in the separable, ill-conditioned, and weakly structured problems, whereas for the moderate functions no impact can be observed and the multi-modal functions f_{15}–f_{19} are not solved by both algorithms. The large difference in the weakly structured problems are only due to the two Gallagher functions—the two problems where already in 2009, the (1+1)-CMA-ES showed better results than the BI-POP-CMA-ES. Here, the number of successes does not or only slightly differs between the (1+1)-CMA-ES and the (1+2_{m}^{s})-CMA-ES, but in case of a success, the (1+2_{m}^{s})-CMA-ES is even faster than the (1+1)-CMA-ES by about 30% (not statistically significant).

5. CONCLUSION

In this paper, we have compared the recently introduced (1+2_{m}^{s})-CMA-ES algorithm with the (1+1)-CMA-ES. The (1+2_{m}^{s})-CMA-ES algorithm beats the best performing algorithm of BBOB-2009 on the rotated ellipsoid function f_{10} with a 17% smaller expected running time. We see that mirroring and sequentialism improve the performance of the (1+1)-CMA-ES by a factor of 10–20% on the unimodal functions f_1, f_2, f_{10}, f_{11} and f_{14}. Moreover, there is no statistically significant result where the (1+2_{m}^{s})-CMA-ES is worse than the (1+1)-CMA-ES.

6. ACKNOWLEDGMENTS

This work received support by the French national research agency (ANR) within the SYSCOMM project ANR-08-SYSC-017 and within the COSINUS project ANR-08-COSI-007-12.

7. REFERENCES

[1] A. Auger, D. Brockhoff, and N. Hansen. Mirrored sampling and sequential selection for evolution strategies. Rapport de Recherche RR-7249, INRIA Saclay—Île-de-France, April 2010.

[2] A. Auger, S. Finck, N. Hansen, and R. Ros. BBOB 2009: Comparison tables of all algorithms on all

[1]We assume minimization.

[2]`http://coco.gforge.inria.fr/doku.php?id=bbob-2010-results`

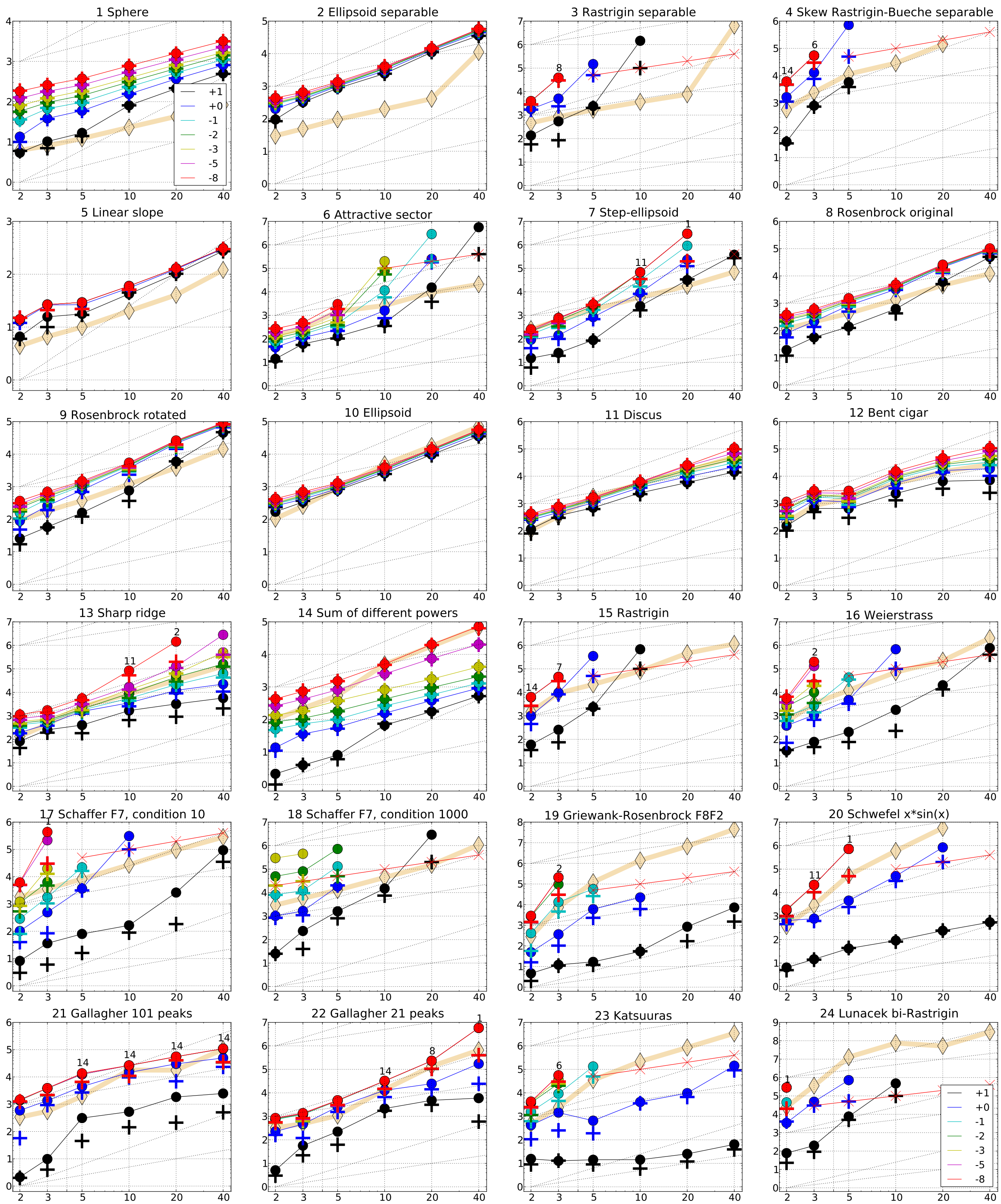

Figure 1: Expected Running Time (ERT, ●) of (1+2_m^s)-CMA-ES to reach $f_{\mathrm{opt}} + \Delta f$ and median number of f-evaluations from successful trials (+), for $\Delta f = 10^{\{+1,0,-1,-2,-3,-5,-8\}}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. For each function and dimension, $\mathrm{ERT}(\Delta f)$ equals to #FEs(Δf) divided by the number of successful trials, where a trial is successful if $f_{\mathrm{opt}} + \Delta f$ was surpassed. The #FEs(Δf) are the total number (sum) of f-evaluations while $f_{\mathrm{opt}} + \Delta f$ was not surpassed in the trial, from all (successful and unsuccessful) trials, and f_{opt} is the optimal function value. Crosses (×) indicate the total number of f-evaluations, #FEs($-\infty$), divided by the number of trials. Numbers above ERT-symbols indicate the number of successful trials. Y-axis annotations are decimal logarithms. The thick light line with diamonds shows the single best results from BBOB-2009 for $\Delta f = 10^{-8}$. Additional grid lines show linear and quadratic scaling.

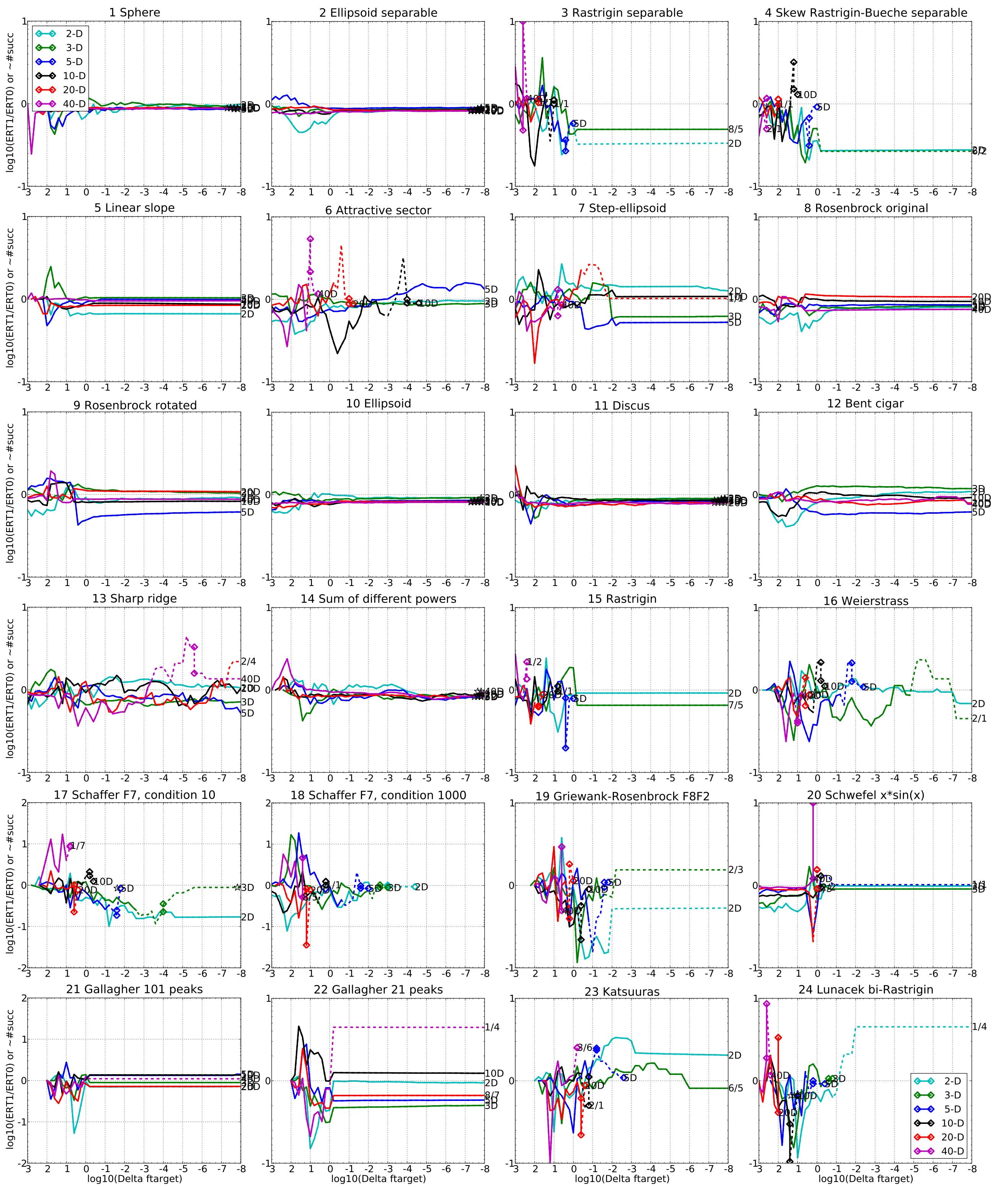

Figure 2: ERT ratio of (1+$2^{\mathrm{s}}_{\mathrm{m}}$)-CMA-ES divided by (1+1)-CMA-ES versus $\log_{10}(\Delta f)$ for f_1–f_{24} in 2, 3, 5, 10, 20, 40-D. Ratios $< 10^0$ indicate an advantage of (1+$2^{\mathrm{s}}_{\mathrm{m}}$)-CMA-ES, smaller values are always better. The line gets dashed when for any algorithm the ERT exceeds thrice the median of the trial-wise overall number of f-evaluations for the same algorithm on this function. Symbols indicate the best achieved Δf-value of one algorithm (ERT gets undefined to the right). The dashed line continues as the fraction of successful trials of the other algorithm, where 0 means 0% and the y-axis limits mean 100%, values below zero for (1+$2^{\mathrm{s}}_{\mathrm{m}}$)-CMA-ES. The line ends when no algorithm reaches Δf anymore. The number of successful trials is given, only if it was in $\{1 \ldots 9\}$ for (1+$2^{\mathrm{s}}_{\mathrm{m}}$)-CMA-ES (1st number) and non-zero for (1+1)-CMA-ES (2nd number). Results are significant with $p = 0.05$ for one star and $p = 10^{-\#\star}$ otherwise, with Bonferroni correction within each figure.

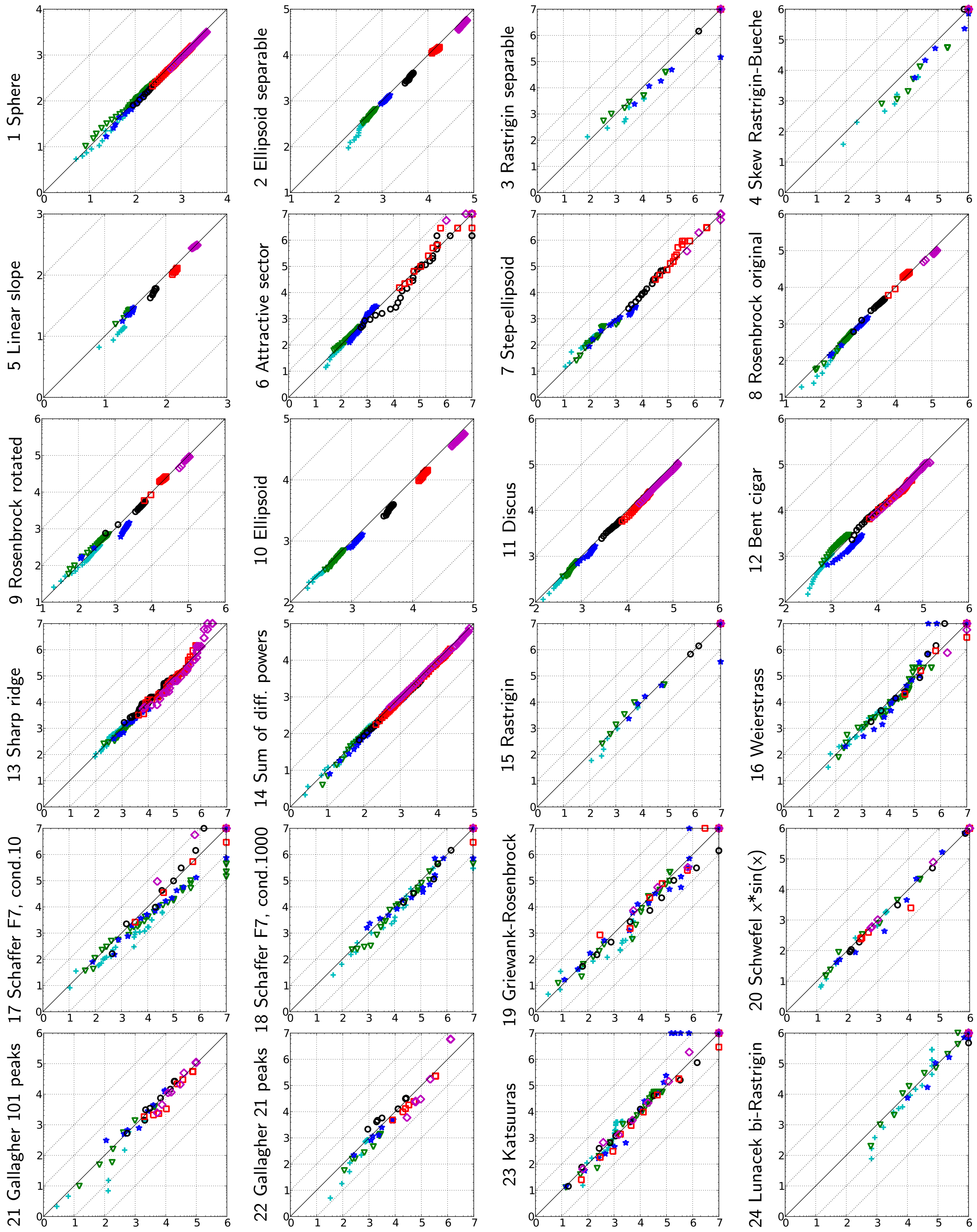

Figure 3: Expected running time (ERT in log10 of number of function evaluations) of $(1+2^{\mathrm{s}}_{\mathrm{m}})$-CMA-ES versus (1+1)-CMA-ES for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_1–f_{24}. Markers on the upper or right edge indicate that the target value was never reached by $(1+2^{\mathrm{s}}_{\mathrm{m}})$-CMA-ES or (1+1)-CMA-ES respectively. Markers represent dimension: 2:$+$, 3:$\triangledown$, 5:$\star$, 10:$\circ$, 20:$\square$, 40:$\diamond$.

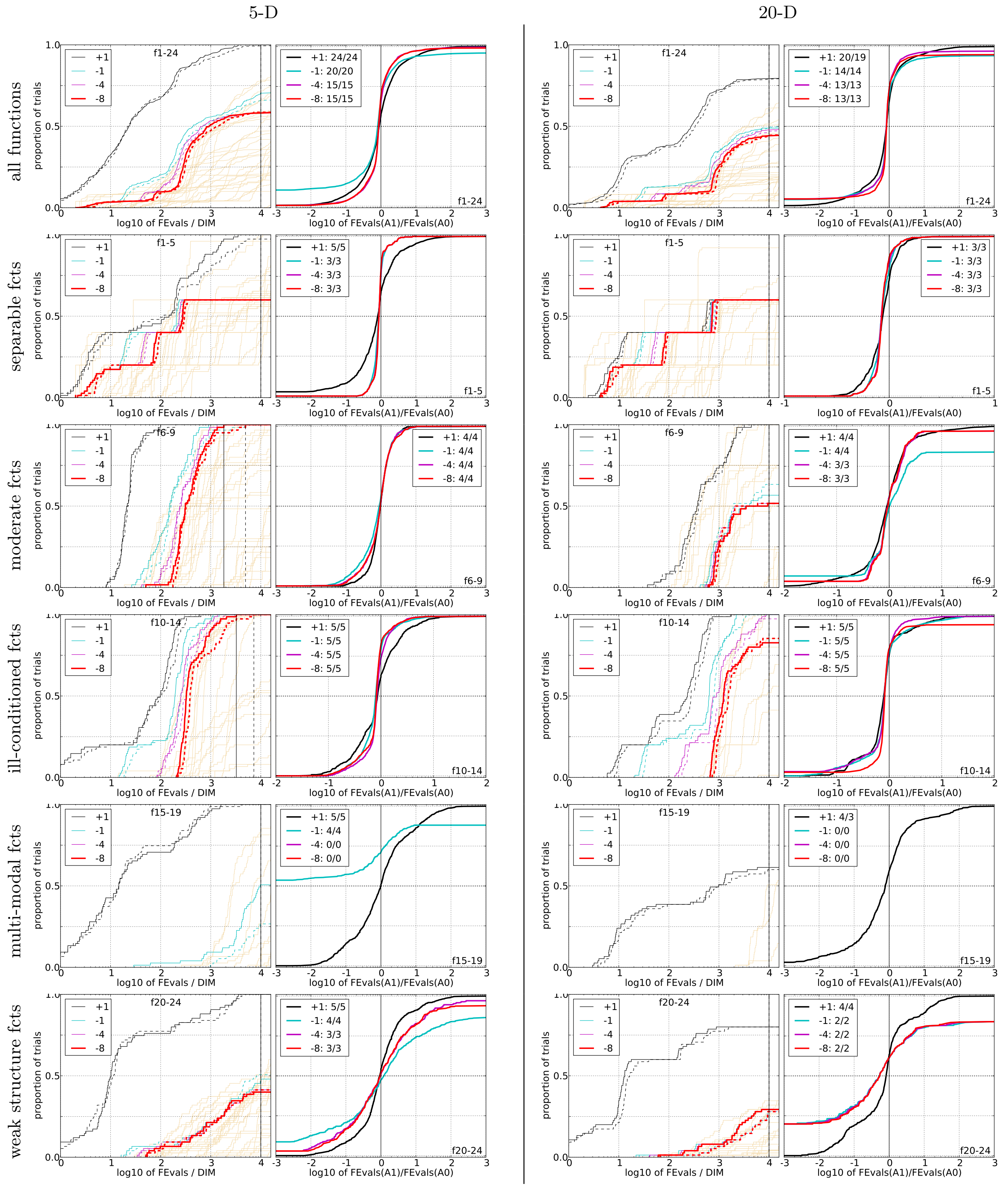

Figure 4: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}}+\Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for (1+$2_{\mathrm{m}}^{\mathrm{s}}$)-CMA-ES (solid) and (1+1)-CMA-ES (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of (1+$2_{\mathrm{m}}^{\mathrm{s}}$)-CMA-ES divided by (1+1)-CMA-ES, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial ((1+$2_{\mathrm{m}}^{\mathrm{s}}$)-CMA-ES first).

5-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_1	11	12	12	12	12	12	15/15
(1+1)-CMA-ES	2.1	5.6	9.6	17	24	32	15/15
$(1+2^{s}_{m})$-CMA-ES	1.5	4.9	8	15	21	**28**$^{\star 2}$	15/15
f_2	83	87	88	90	92	94	15/15
(1+1)-CMA-ES	11	13	13	14	15	15	15/15
$(1+2^{s}_{m})$-CMA-ES	10	11	12	13	13	14	15/15
f_3	720	1600	1600	1600	1700	1700	15/15
(1+1)-CMA-ES	7.2	∞	∞	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	3.4	91	∞	∞	∞	∞ *5.0e4*	0/15
f_4	810	1600	1700	1800	1900	1900	15/15
(1+1)-CMA-ES	22	∞	∞	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	7.1	440	∞	∞	∞	∞ *5.0e4*	0/15
f_5	10	10	10	10	10	10	15/15
(1+1)-CMA-ES	2	2.9	3	3	3	3	15/15
$(1+2^{s}_{m})$-CMA-ES	1.8	2.7	2.9	2.9	2.9	2.9	15/15
f_6	110	210	280	580	1000	1300	15/15
(1+1)-CMA-ES	1.8	1.5	1.9	1.4	1.1	1.4	15/15
$(1+2^{s}_{m})$-CMA-ES	1.1	1.2	1.6	1.4	1.6	2.2	15/15
f_7	24	320	1200	1600	1600	1600	15/15
(1+1)-CMA-ES	4.1	2.8	3.2	3.6	3.6	3.5	15/15
$(1+2^{s}_{m})$-CMA-ES	3.7	2.6	1.4	1.9	1.9	1.8	15/15
f_8	73	270	340	390	410	420	15/15
(1+1)-CMA-ES	2.3	3.5	3.8	3.8	4	4.1	15/15
$(1+2^{s}_{m})$-CMA-ES	1.9	3	3.2	3.3	3.4	3.5	15/15
f_9	35	130	210	300	340	370	15/15
(1+1)-CMA-ES	3.3	13	9	7.2	6.8	6.4	15/15
$(1+2^{s}_{m})$-CMA-ES	4.6	6.2	5	4.2	4	3.9	15/15
f_{10}	350	500	570	630	830	880	15/15
(1+1)-CMA-ES	2.6	2.1	2.1	2	1.7	1.7	15/15
$(1+2^{s}_{m})$-CMA-ES	2.2	1.7	**1.7**$^{\star 2}$	**1.7**$^{\star 2}$	**1.4**$^{\star 2}$	**1.4**$^{\star 2}$	15/15
f_{11}	140	200	760	1200	1500	1700	15/15
(1+1)-CMA-ES	5.8	6.9	2.1	1.5	1.3	1.2	15/15
$(1+2^{s}_{m})$-CMA-ES	4.9	5.2	**1.7***	**1.2**$^{\star 3}$	**1.1**$^{\star 2}$	**0.98**$^{\star 2}$	15/15
f_{12}	110	270	370	460	1300	1500	15/15
(1+1)-CMA-ES	7.8	6.8	6.9	6.9	3.1	3	15/15
$(1+2^{s}_{m})$-CMA-ES	6	4.2	4	4.2	1.8	1.8	15/15
f_{13}	130	190	250	1300	1800	2300	15/15
(1+1)-CMA-ES	3.7	7.1	8.3	2.4	2.2	3.1	15/15
$(1+2^{s}_{m})$-CMA-ES	3.1	7.9	7.1	1.9	2	2.3	15/15
f_{14}	9.8	41	58	140	250	480	15/15
(1+1)-CMA-ES	1.2	1.8	2.3	2.7	4.1	3.3	15/15
$(1+2^{s}_{m})$-CMA-ES	0.82	1.4	1.7	2.7	3.2	**2.7**$^{\star 2}$	15/15
f_{15}	510	9300	1.9e4	2.0e4	2.1e4	2.1e4	14/15
(1+1)-CMA-ES	6.1	∞	∞	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	4.6	38	∞	∞	∞	∞ *5.0e4*	0/15
f_{16}	120	610	2700	1.0e4	1.2e4	1.2e4	15/15
(1+1)-CMA-ES	2	21	19	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	1.7	7.7	16	∞	∞	∞ *5.0e4*	0/15
f_{17}	5.2	210	900	3700	6400	7900	15/15
(1+1)-CMA-ES	15	25	94	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	15	17	24	∞	∞	∞ *5.0e4*	0/15
f_{18}	100	380	4000	9300	1.1e4	1.2e4	15/15
(1+1)-CMA-ES	8.6	52	87	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	15	48	33	∞	∞	∞ *5.0e4*	0/15
f_{19}	1	1	240	1.2e5	1.2e5	1.2e5	15/15
(1+1)-CMA-ES	13	5.1e3	1.5e3	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	17	6.0e3	230	∞	∞	∞ *5.0e4*	0/15
f_{20}	16	850	3.8e4	5.4e4	5.5e4	5.5e4	14/15
(1+1)-CMA-ES	2.8	10	19	13	13	13	1/15
$(1+2^{s}_{m})$-CMA-ES	2.6	5.4	19	13	13	13	1/15
f_{21}	41	1200	1700	1700	1700	1800	14/15
(1+1)-CMA-ES	2.7	3.4	5.6	5.5	5.5	5.4	15/15
$(1+2^{s}_{m})$-CMA-ES	7.6	3.8	7.5	7.4	7.3	7.2	14/15
f_{22}	71	390	940	1000	1000	1100	14/15
(1+1)-CMA-ES	3.7	8	8.5	8	7.9	7.8	15/15
$(1+2^{s}_{m})$-CMA-ES	3.1	6.5	4.9	4.6	4.6	4.5	15/15
f_{23}	3	520	1.4e4	3.2e4	3.3e4	3.4e4	15/15
(1+1)-CMA-ES	5	5.4	5.5	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	4.7	1.3	9.3	∞	∞	∞ *5.0e4*	0/15
f_{24}	1600	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
(1+1)-CMA-ES	6.2	∞	∞	∞	∞	∞ *5.0e4*	0/15
$(1+2^{s}_{m})$-CMA-ES	4.7	3.3	∞	∞	∞	∞ *5.0e4*	0/15

20-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_1	43	43	43	43	43	43	15/15
(1+1)-CMA-ES	5.5	9.6	14	21	29	37	15/15
$(1+2^{s}_{m})$-CMA-ES	4.9	8.5	12	19	**26***	**33**$^{\star 2}$	15/15
f_2	380	390	390	390	390	390	15/15
(1+1)-CMA-ES	32	38	41	43	44	45	15/15
$(1+2^{s}_{m})$-CMA-ES	29	**33***	**35**$^{\star 2}$	**36**$^{\star 3}$	**37**$^{\star 3}$	**37**$^{\star 3}$	15/15
f_3	5100	7600	7600	7600	7600	7700	15/15
(1+1)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
f_4	4700	7600	7700	7700	7800	1.4e5	9/15
(1+1)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
f_5	41	41	41	41	41	41	15/15
(1+1)-CMA-ES	3.1	3.6	3.8	3.8	3.8	3.8	15/15
$(1+2^{s}_{m})$-CMA-ES	2.5	3.1	3.2	3.2	3.2	3.2	15/15
f_6	1300	2300	3400	5200	6700	8400	15/15
(1+1)-CMA-ES	13	92	830	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	12	110	850	∞	∞	∞ *2.0e5*	0/15
f_7	1400	4300	9500	1.7e4	1.7e4	1.7e4	15/15
(1+1)-CMA-ES	24	41	37	180	180	170	1/15
$(1+2^{s}_{m})$-CMA-ES	24	54	96	180	180	180	1/15
f_8	2000	3900	4000	4200	4400	4500	15/15
(1+1)-CMA-ES	3.2	4.8	5.1	5.3	5.3	5.3	15/15
$(1+2^{s}_{m})$-CMA-ES	3	5.4	5.6	5.7	5.7	5.7	15/15
f_9	1700	3100	3300	3500	3600	3700	15/15
(1+1)-CMA-ES	3.6	6.2	6.4	6.5	6.5	6.5	15/15
$(1+2^{s}_{m})$-CMA-ES	3.4	6.9	7.1	7.1	7.1	7	15/15
f_{10}	7400	8700	1.1e4	1.5e4	1.7e4	1.7e4	15/15
(1+1)-CMA-ES	1.7	1.7	1.4	1.1	0.99	0.99	15/15
$(1+2^{s}_{m})$-CMA-ES	**1.3***	**1.3**$^{\star 2}$	**1.2**$^{\star 3}$	**0.91**$^{\star 3 \downarrow}$	**0.82**$^{\star 3 \downarrow 4}$	**0.82**$^{\star 3 \downarrow 4}$	15/15
f_{11}	1000	2200	6300	9800	1.2e4	1.5e4	15/15
(1+1)-CMA-ES	7.7	5.9	2.7	2.6	2.4	2.1	15/15
$(1+2^{s}_{m})$-CMA-ES	**5.8***	**4.3***	**2.1***	**2**$^{\star 2}$	**1.9**$^{\star 2}$	**1.7**$^{\star 3}$	15/15
f_{12}	1000	1900	2700	4100	1.2e4	1.4e4	15/15
(1+1)-CMA-ES	6.6	9.7	10	9.1	3.6	3.7	15/15
$(1+2^{s}_{m})$-CMA-ES	6.3	8.3	8.2	6.9	2.9	3.1	15/15
f_{13}	650	2000	2800	1.9e4	2.4e4	3.0e4	15/15
(1+1)-CMA-ES	6.5	4.8	10	4.1	7.6	13	4/15
$(1+2^{s}_{m})$-CMA-ES	4.9	6	7.7	3.3	5.2	18	2/15
f_{14}	75	240	300	930	1600	1.6e4	15/15
(1+1)-CMA-ES	2.9	1.9	2.3	2.2	5.4	1.2	15/15
$(1+2^{s}_{m})$-CMA-ES	2.3	1.6	**1.9**$^{\star 2}$	1.9	4.5	**0.97***	15/15
f_{15}	3.0e4	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
(1+1)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
f_{16}	1400	2.7e4	7.7e4	1.9e5	2.0e5	2.2e5	15/15
(1+1)-CMA-ES	31	∞	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	15	∞	∞	∞	∞	∞ *2.0e5*	0/15
f_{17}	63	1000	4000	3.1e4	5.6e4	8.0e4	15/15
(1+1)-CMA-ES	53	∞	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	42	∞	∞	∞	∞	∞ *2.0e5*	0/15
f_{18}	620	4000	2.0e4	6.8e4	1.3e5	1.5e5	15/15
(1+1)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	4.7e3	∞	∞	∞	∞	∞ *2.0e5*	0/15
f_{19}	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
(1+1)-CMA-ES	290	2.9e6	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	850	∞	∞	∞	∞	∞ *2.0e5*	0/15
f_{20}	82	4.6e4	3.1e6	5.5e6	5.6e6	5.6e6	14/15
(1+1)-CMA-ES	3.3	20	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	2.9	18	∞	∞	∞	∞ *2.0e5*	0/15
f_{21}	560	6500	1.4e4	1.5e4	1.6e4	1.8e4	15/15
(1+1)-CMA-ES	3.5	5.5	5.4	5.2	4.9	4.4	14/15
$(1+2^{s}_{m})$-CMA-ES	3.3	4.6	3.9	3.8	3.6	3.2	14/15
f_{22}	470	5600	2.3e4	2.5e4	2.7e4	1.3e5	12/15
(1+1)-CMA-ES	17	9.4	15	14	13	2.6	7/15
$(1+2^{s}_{m})$-CMA-ES	10	4.4	9.8	9.2	8.6	1.7	8/15
f_{23}	3.2	1600	6.7e4	4.9e5	8.1e5	8.4e5	15/15
(1+1)-CMA-ES	18	8.1	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	7.9	6	∞	∞	∞	∞ *2.0e5*	0/15
f_{24}	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
(1+1)-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15
$(1+2^{s}_{m})$-CMA-ES	∞	∞	∞	∞	∞	∞ *2.0e5*	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for (1+1)-CMA-ES and $(1+2^{s}_{m})$-CMA-ES for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

noiseless functions. Technical Report RT-0383, INRIA, April 2010.

[3] A. Auger and N. Hansen. Benchmarking the (1+1)-CMA-ES on the BBOB-2009 function testbed. In Rothlauf [13], pages 2459–2466.

[4] A. Auger and N. Hansen. Benchmarking the (1+1)-CMA-ES on the BBOB-2009 noisy testbed. In Rothlauf [13], pages 2467–2472.

[5] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009. Updated February 2010.

[6] N. Hansen. Benchmarking a BI-population CMA-ES on the BBOB-2009 function testbed. In Rothlauf [13], pages 2389–2396.

[7] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2010: Experimental setup. Technical Report RR-7215, INRIA, 2010.

[8] N. Hansen, A. Auger, R. Ros, S. Finck, and P. Pošík. Comparing results of 31 algorithms from the black-box optimization benchmarking BBOB-2009. In *Workshop Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2010)*. ACM Press, 2010. to appear.

[9] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009. Updated February 2010.

[10] N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation*, 9(2):159–195, 2001.

[11] C. Igel, T. Suttorp, and N. Hansen. A computational efficient covariance matrix update and a (1+1)-CMA for evolution strategies. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2006)*, pages 453–460. ACM Press, 2006.

[12] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE International Congress on Evolutionary Computation*, pages 153–157, 1997.

[13] F. Rothlauf, editor. *Genetic and Evolutionary Computation Conference, GECCO 2009, Proceedings, Montreal, Québec, Canada, July 8-12, 2009, Companion Material*. ACM, 2009.